

# Bulk Viscosity in SU(2) Gauge Theory

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Understanding QGP through Spectral Functions and Euclidean  
Correlators

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# Introduction

- Bulk viscosity may become large close to  $T_c$ , thus highly relevant for the QGP
  - D. Kharzeev and K. Tuchin, arXiv:0705.4280 [hep-ph]
  - F. Karsch, D. Kharzeev and K. Tuchin, arXiv:0711.0914 [hep-ph]
  - H. B. Meyer, arXiv:0710.3717 [hep-lat]
- SU(2) gauge theory ideal model to explore **critical behavior** of transport coefficients
- Compare known properties of the 3d Ising universality class to transport properties of QFTs in 3 + 1 dim
- **Chiral critical point** of QCD also belongs to 3d Ising universality class
- **today**: analyze relation between correlation functions of the energy-momentum tensor and universal behavior of bulk thermodynamics; **no spectral functions**; lattices with small  $N_\tau$  sufficient

# Observables

- Energy-momentum tensor:

$$\Theta^{\mu\mu} = \left( \frac{T^2}{V} \frac{\partial}{\partial T} - 3T \frac{\partial}{\partial V} \right) \ln Z(T, V) = \frac{T}{V} \int_0^{1/T} d\tau \int_V d^3x \langle \Theta^{\mu\mu}(\vec{x}, \tau) \rangle_T$$

- its fluctuation:

$$\begin{aligned}\delta\Theta^{\mu\mu} &= \left( \frac{T^2}{V} \frac{\partial}{\partial T} - 3T \frac{\partial}{\partial V} \right)^2 \ln Z(T, V) \\ &= T \int_0^{1/T} d\tau \int_V d^3x \langle \Theta^{\mu\mu}(\vec{x}, \tau) \Theta^{\mu\mu}(\vec{0}, 0) \rangle_T^c \\ &= T (T c_V - 3T s - 4(\epsilon - 3P))\end{aligned}$$

- in SU(2): fluctuations diverge like **specific heat**  $T^2 c_V \propto t^{-\alpha}$ ,  $t = \frac{|T-T_c|}{T_c}$

# Observables II

- local energy-momentum tensor on the lattice (std Wilson action):

$$\Theta^{\mu\mu}(x) = B(g) \sum_{\rho > \nu} 1 - \frac{1}{N_c} \text{Re Tr } P_{\rho\nu}(x), \quad B(g) \text{ is the } \beta\text{-function}$$

- Correlation function connected to the bulk viscosity  $\zeta$ :

$$G_\zeta(\tau, T) = \int d^3x \langle \Theta_{\mu\mu}(\vec{x}, \tau) \Theta_{\mu\mu}(\vec{0}, 0) \rangle_T^c \quad \text{with} \quad \tau \in [0, 1/T)$$

- spectral function at vanishing momentum:

$$G_\zeta(\tau, T) = \int_0^\infty d\omega \rho_\zeta(\omega, T) K(\tau, \omega, T) \quad \text{with} \quad K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh(\frac{\omega}{2T})}$$

- Bulk viscosity  $\zeta(T)$  through **Kubo-formula**:

$$\zeta(T) = \frac{\pi}{9} \lim_{\omega \rightarrow 0} \frac{\rho_\zeta(\omega, T)}{\omega}$$

# Scaling

- for  $T \rightarrow T_c^\pm$ :

$$G_\zeta(\tau, T) \sim \delta\Theta^{\mu\mu} \sim T^2 c_V \sim A_\pm \left| \frac{T - T_c}{T_c} \right|^{-\alpha} \left( 1 + B_\pm \left| \frac{T - T_c}{T_c} \right|^\Delta + \dots \right)$$

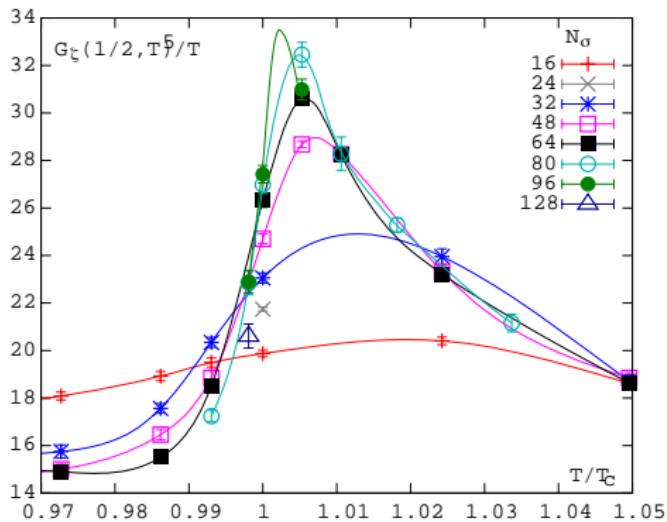
- $\alpha, \Delta, \nu, g_A = A_+/A_-$  from 3d Ising universality class

A. Pelissetto and E. Vicari, Phys. Rept. **368**, 549 (2002)

$\alpha$	$\nu$	$\Delta/\nu$	$g_A = A_+/A_-$
0.110(1)	0.6301(4)	0.84(4)	0.54(1)

# Volume Scaling

- use standard Wilson action, heatbath/overrelaxation algorithms
- $\beta_c(N_\tau = 4) = 2.29895(10)$  J. Engels and T. Scheideler, Nucl. Phys. B 539, 557 (1999)
- lattice sizes  $N_\sigma^3 \times 4$ , with  $N_\sigma = 32, \dots, 128$
- statistics:  $4 \cdot 10^5$  to  $1 \cdot 10^6$  configurations
- $G_\zeta(\tau T = 1/2, T, N_\sigma)$ :



# Ratio

- don't attempt to extract  $\rho_{\text{sing}}$  from  $G_\zeta$  here, since  $N_\tau = 4$ -data only
- split spectral function:

$$\rho_\zeta(\omega, T) = \rho_{\text{sing}}(\omega, T) + \rho_{\text{reg}}(\omega, T)$$

- $\rho_{\text{sing}}$  should be linear in  $\omega$

$$\rho_{\text{sing}}(\omega, T) \propto \omega f(\omega, \omega_0(T)) \quad \text{with} \quad f(0, \omega_0(T)) = \frac{9}{\pi}$$

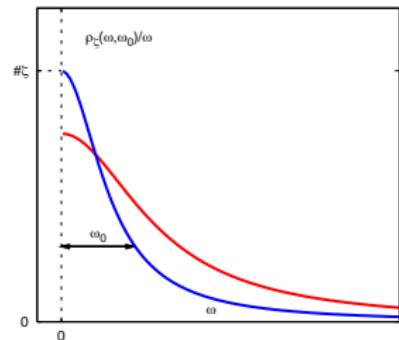
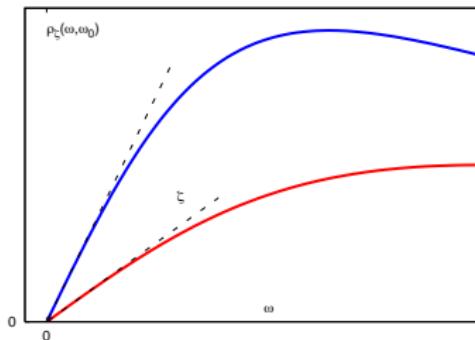
- test structure of  $f(\omega, \omega_0(T))$  by:

$$R(\delta t, T) = \frac{G_\zeta(1/2T + \delta t, T)}{G_\zeta(1/2T, T)} \quad \text{and} \quad \Delta G_\zeta = G_\zeta(1/2T + \delta t, T) - G_\zeta(1/2T, T)$$

- $\Delta G_\zeta$  eliminates contribution from  $\omega = 0$
- $R(\delta t, T) = 1$  means that  $\rho(\omega = 0) \rightarrow \infty$

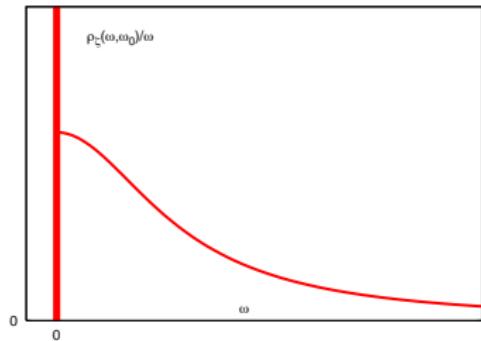
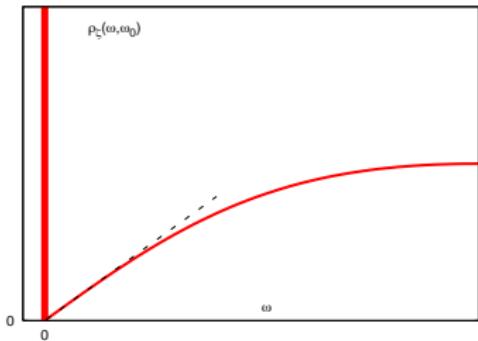
# Ratio II

- Ansatz for  $\rho_\zeta(\omega)$ , e. g.  $\rho_\zeta(\omega) = \zeta(T) \omega f(\omega, \omega_0)$  with  $f(\omega, \omega_0)$  Breit-Wigner-fct
- D. Kharzeev and K. Tuchin, arXiv:0705.4280 [hep-ph]  
F. Karsch, D. Kharzeev and K. Tuchin, arXiv:0711.0914 [hep-ph]



# Ratio II

- $\rho_\zeta(\omega)$  might contain  $c(T) \omega \delta(\omega)$



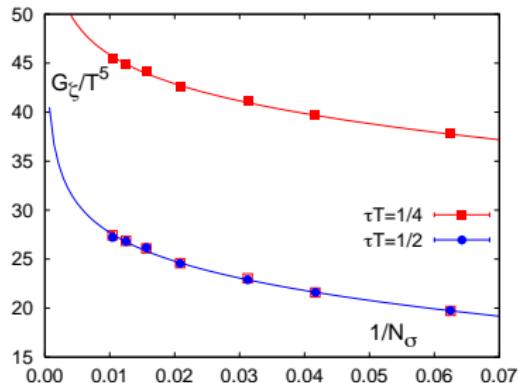
# Finite Size Scaling at $T_c$

- Fit data with ansatz, parameters:  $A_\sigma, B_\sigma, C_\sigma$ , may depend on  $\tau$ :

$$G_\zeta(\hat{\tau}, T_c)/T_c^5 = A_\sigma N_\sigma^{\alpha/\nu} \left( 1 + B_\sigma N_\sigma^{-\Delta/\nu} \right) + C_\sigma$$

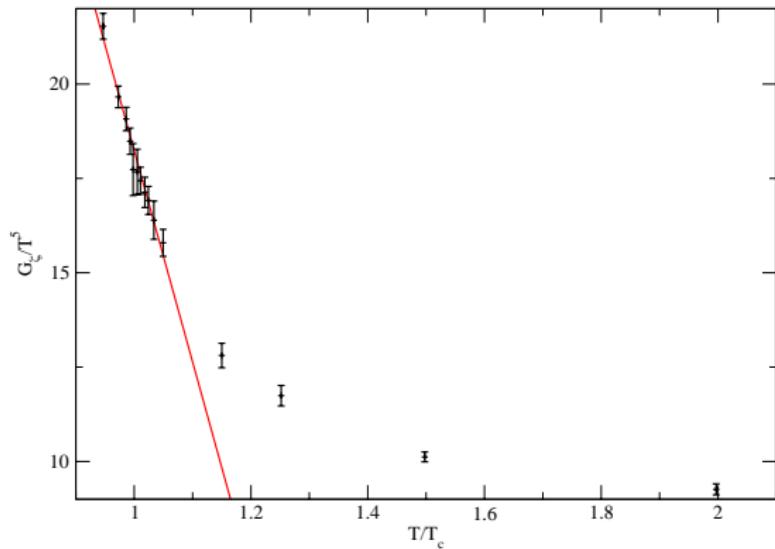
- critical behavior that of 3d Ising
- $G_\zeta(\tau, T_c)$  independent of  $\tau$
- $R(\delta t, T_c) = 1$ , thus  $\rho_\zeta/\omega$  has  $\delta$ -fct-like singularity at  $T_c$

$\tau T$	$A_\sigma$	$B_\sigma$	$C_\sigma$
1/4	9.2(1.2)	-2.4(1.0)	26.3(2.7)
1/2	9.1(1.2)	-2.4(0.9)	8.4(2.9)
combined fit	9.15(73)	-2.39(59)	26.4(1.9) ( $\tau T = 1/4$ ) 8.3(1.7) ( $\tau T = 1/2$ )



# Background

- $\Delta G_\zeta = G_\zeta(\tau T = 1/4) - G_\zeta(\tau T = 1/2)$
- singular part indeed eliminated

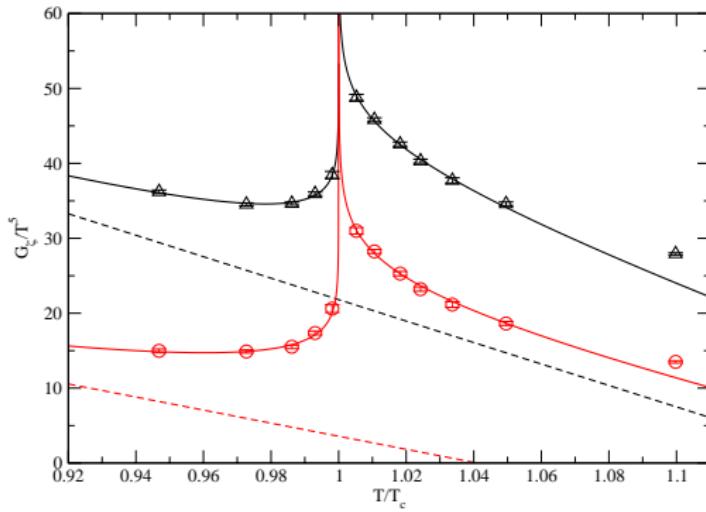


# Scaling with $T$

- Fit  $T$ -dependence with ansatz, parameters  $A_+, B_{\pm}, C, D$ , all **may depend on  $\tau$** :

$$G_\zeta(\hat{\tau}, T)/T^5 = A_{\pm} t^{-\alpha} (1 + B_{\pm} t^\Delta) + C + Dt$$

- fit range:  $T/T_c \in [0.94, 1.05]$
- $A_+, B_{\pm}$  agree for both  $\tau T$  within errors and with combined fit



# Fit data

- Scaling with  $T$ :  $A_+, B_{\pm}$  agree within errors for both  $\tau T$  and with combined fit,  $C, D$  same for corresponding  $\tau T$
- Scaling at  $T_c$ :  $A_\sigma, B_\sigma, C_\sigma$  agree within errors for both  $\tau T$  and with combined fit
- Background:  $\Delta C$  and  $\Delta C_\sigma$  close
- Fit results:

	$\tau T$	$A_+$	$B_+$	$B_-$	$C$	$D$	$A_\sigma$	$B_\sigma$	$C_\sigma$
free fit	1/4	16.3(2.1)	-0.59(19)	-1.9(1.4)	21.5(5.7)	-150(53)	9.2(1.2)	-2.4(1.0)	26.3(2.7)
	1/2	16.4(2.2)	-0.76(18)	-2.2(1.5)	3.7(3.4)	-95(32)	9.1(1.2)	-2.4(0.9)	8.4(2.9)
combined fit	1/4	16.5(1.3)	-0.74(10)	-2.12(86)	21.8(2.1)	-143(20)	9.15(73)	-2.39(59)	26.4(1.9)
	1/2				3.8(1.9)	-87(18)			8.3(1.7)

→ no  $\tau$ -dependence: singularity arises from exact  $\delta$ -function or requires small  $\omega_0$

# Results so far

- $G_\zeta(\tau T, T) \sim \delta\theta^{\mu\mu} \sim T^2 c_V \sim t^{-\alpha}$
- $\rho_\zeta/\omega$  has a  $\delta$ -function-like singularity at  $\omega = 0$  at  $T_c$
- current results are consistent with a  $\delta$ -function contribution at  $\omega = 0$  as well as a diverging bulk viscosity for  $T \rightarrow T_c$
- if  $\zeta$  diverges, its divergence is stronger than that of the specific heat and  $\omega_0$  has to vanish

# Bulk Viscosity I

- assume Breit-Wigner-Ansatz for spectral density (no  $\delta$ -function):

$$\rho_{\text{sing}}(\omega, \omega_0) = \zeta(T) \omega f(\omega, \omega_0) \quad \text{with} \quad f(\omega, \omega_0) = \frac{9}{\pi} \frac{\omega_0^2}{\omega^2 + \omega_0^2}$$

- Correlator:

$$\begin{aligned} G_{\zeta}^{\text{sing}}(\tau T, T) &= \frac{9}{\pi} \zeta(T) \omega_0(T) \int dx \frac{\omega_0 x}{1+x^2} \frac{\cosh(\omega_0 x(\tau - 1/2T))}{\sinh(\omega_0 x/2T)} \\ &= 9T\zeta(T)\omega_0(T) \left( 1 - \frac{\omega_0(T)}{2T} \ln [2 - 2\cos(2\pi T\tau)] + \mathcal{O}(\omega_0^2) \right) \end{aligned}$$

- bulk viscosity:

$$\frac{\zeta}{T^3} = \frac{1}{9} \left( \frac{\omega_0}{T} \right)^{-1} t^{-\alpha} \begin{cases} A_+ (1 - B_+ t^\Delta) & T > T_c \\ A_- (1 - B_- t^\Delta) & T < T_c \end{cases}$$

# Bulk Viscosity II

- estimate  $\omega_0$  by **dynamic scaling relation**

$$\frac{\omega_0}{T} = \xi^{-z} \sim t^{z\nu}$$

- dynamical critical exponent:**  $z \sim 1, \dots, 2, \dots, 3$

- ▶ liquid-gas-transitions have  $z \sim 3$ , thus  $\zeta \sim t^{-2}$
- ▶ 3d Ising spin relaxation gives  $z \sim 2$
- ▶ conservative:  $z \sim 1$

P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**, 435 (1977)

- correlation length  $\xi$  from Polyakov loop correlators

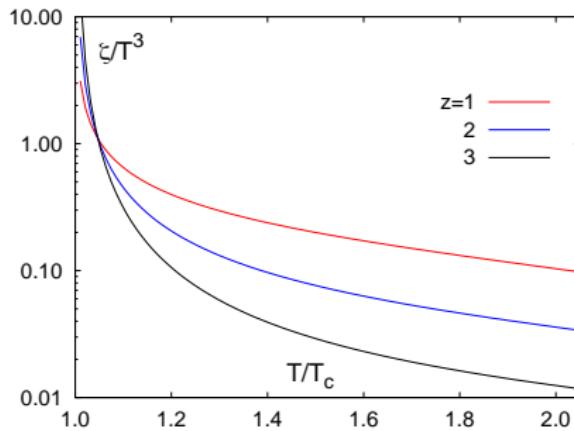
S. Digal, S. Fortunato and P. Petreczky, Phys. Rev. D **68**, 034008 (2003)

# Bulk Viscosity III

- estimate  $\zeta/T^3 = 0.01 - 0.1$  at  $2T_c$ , rises by  $O(10)$  when going to  $1.2T_c$
- convert to  $\zeta/s$  via  $s/T^3 \simeq 0.3$  at  $T_c$  and  $s/T^3 \simeq 2$  at  $2T_c$

J. Engels, F. Karsch, H. Satz and I. Montvay, Nucl. Phys. B **205**, 545 (1982)

J. Engels, F. Karsch and K. Redlich, Nucl. Phys. B **435**, 295 (1995)



# Conclusions and Outlook

- studied euclidean-time correlators of the energy momentum tensor in 3+1 dim SU(2) gauge theory on lattices with  $N_\tau = 4$
- $G_\zeta$  diverges at  $T_c$  with the **3d Ising** critical exponent  $\alpha$
- $\rho_\zeta$  has  $\delta$ -function-like **singularity** at  $T_c$
- bulk viscosity  $\zeta$  may **diverge** stronger than  $\alpha$ ,  $\omega_0$  may **vanish** at  $T_c$
- want to use larger  $N_\tau$ -lattices to control  $\tau$ -dependence of  $G_\zeta$  and to extract spectral functions, determine  $\zeta$ ,  $\omega_0$  and  $\eta$  etc.

# Shear Viscosity

- $G_\eta(\tau, T) = \sum_{i < j} \int d\mathbf{x} \langle \theta_{ij}(\mathbf{x}, \tau) \theta_{ij}(\mathbf{0}, 0) \rangle_T^c$

- $\eta \sim t^{-\nu x}$  with  $x \sim 0.05$

P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**, 435 (1977)

